Combination of Resistances

Objectives

After going through this module ,the students will be able to:

- Identify carbon resistance and know its value using colour coding

- Know wire resistances, their nature and types.

- Know about series and parallel grouping of resistances

- Make combinations with resistances for net equivalent resistance for useful purposes

Content Outline

- Unit Syllabus

- Module Wise Distribution of unit syllabus

- Words you must know

- Introduction

- Role of Resistance

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- Need for Series and Parallel Combinations of Resistances.

- Types of Resistors

- Color Coding of Carbon Resistances

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Unit Syllabus

Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility, and their relation with electric current; Ohm's law, electrical resistance, V-I characteristics (linear and nonlinear), electrical energy and power, electrical resistivity and conductivity.

Carbon resistors, colour code for carbon resistors; series & parallel combinations of resistors; temperature dependence of resistance.

Internal resistance of a cell, Potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and simple applications; Wheatstone bridge, metre bridge.

Potentiometer- principle and its application to measure potential difference and for comparing emf of two cells; measurement of internal resistance of a cell.

Module Wise Distribution - 08 Modules

The above unit has been divided into 8 modules for better understanding.

Module 1	- Electric current, - Solids liquids and gases - Need for charge carriers speed of charge carriers in a metallic conductor - Flow of electric charges in a metallic conductor - Drift velocity, - Mobility and their relation with electric current - Ohm's law
Module 2	- Electrical resistance, - V-I characteristics (linear and nonlinear), - Electrical energy and power, - Electrical resistivity and conductivity - Temperature dependence of resistance
Module 3	- Carbon resistors, - Colour code for carbon resistors; - Metallic Wire resistances - Series and parallel combinations of resistors - Grouping of resistances - Current and potential differences in series and parallel circuits
Module 4	- Internal resistance of a cell, - Potential difference and emf of a cell, - Combination of cells in series and in parallel. - Need for combination of cells
Module 5	- Kirchhoff's Rules - Simple applications. of Kirchhoff's Rules for calculating current s and voltages - Numerical
Module 6	Wheatstone bridge

	● Balanced Wheatstone bridge condition derivation using Kirchhoff's Rules ● Wheatstone bridge and Metre Bridge. ● Application of meter bridge
Module 7	● Potentiometer – ● Principle ● Applications to ● Measure potential difference ● Comparing emf of two cells; ● Measurement of internal resistance of a cell. ● Numerical
Module 8	● Numerical ● Electrical energy and power

Module 3

Words You Must Know

- **Resistance(R)**: In simple words, a 'resistance' can be anything which opposes / obstructs the flow of current. The free electrons (in a conductor) constantly collide with its atoms / ions. These collisions are the basic cause of its 'resistance' to 'current flow'. It is measured as potential difference across a conductor per unit current flowing through it.

$$\frac{V}{I} = R$$

- **Ohm's law**: It states that the current flowing through a given conductor is directly proportional to potential difference across the two ends of the conductor, provided its temperature and other physical conditions remain constant.

$$\frac{V}{I} = R$$

  Here R is the resistance of the conductor, which is constant for a given conductor, under given physical conditions.

- **Potential Difference (V):** It equals the work done in carrying a unit positive charge from one point to another.

$$V = \frac{W}{q}$$

From Ohm's law this is also given as

$$V = IR$$

Introduction

You must have observed that the small decorative electric bulbs used for festive occasions, say for Diwali or marriage decoration, are connected in a row. If one of the bulbs gets fused, the rest of the bulbs stop working.

On the other hand, if a bulb gets 'fused' in a household, the other bulbs/ devices are unaffected. Ever thought why is it so?

It is because, in the first case, the bulbs are connected in series. But in households the bulbs are connected in parallel.

Resistance: It is the measure of opposition in the path of flow of charge. Higher the resistance, smaller is the current flow.

Effect of resistance on current

As resistance is the obstruction in the path of flow of current, it follows that higher the resistance, lower the value of current. It can also be seen from Ohm's law

$$V = IR$$

$$\text{At constant} \quad I \propto 1/R$$

Each electrical device has some resistance of its own, so we can treat each device as a 'resistance'.

Electric Potential

In fluids, pressure difference between the two ends of the tube, determines the rate of flow of the fluid. In electricity, it is the potential difference (V) between the two ends of a given conductor that determines the rate of flow of charge (current) through it. We can think of 'potential' as being similar to 'pressure'. Also 'Potential difference' is the cause of current.

Resistance

Resistance is basically an indicator of the 'obstruction' in the path of flow of anything. In electricity, it is a 'cause' that is hindering the flow of electric charges (current).

Role of Resistance

It can control the amount of flow of charges (electric current). We can think in terms of a 'drop in potential' when electric current flows through a resistor.

Let us consider an example:

If we put a mesh in front of a gas emanating fire fighter jet, the mesh acts as a 'resistance' because it obstructs the flow of gas.

Here, on the incoming side, there is a higher pressure (potential) and at the outgoing side of obstruction (resistance), there is a lower pressure (potential). There is a fall of pressure (potential) across the mesh (resistance).

A similar thing happens in case of electric resistance in electricity. Let us consider current I flows through a resistance R as shown in the figure below.

The incoming end has a potential of say, 200V; the outgoing end may have a potential of, say, 120V.

● There is a potential drop of 80V. As per Ohm's law (V=IR), V the potential difference between the two ends of resistance, equals 80V.

● It is for this reason that we always say there is a "**potential drop across resistance**".

Resistances in electrical circuits can be used to reduce the available voltage or current present in a circuit.

## Misconceptions about Resistances

- Resistances are conductors and not insulators
- An insulator has indefinite resistance
- Resistance wires are straight and not zig zag , in circuit diagrams we represent them by a zig zag line

- Resistances do not have any positive or negative.
- They are neutral, however they may develop a potential gradient across them when connected to a battery.
- You can connect the resistance in any way you like, which means suppose a resistance R has ends P and Q
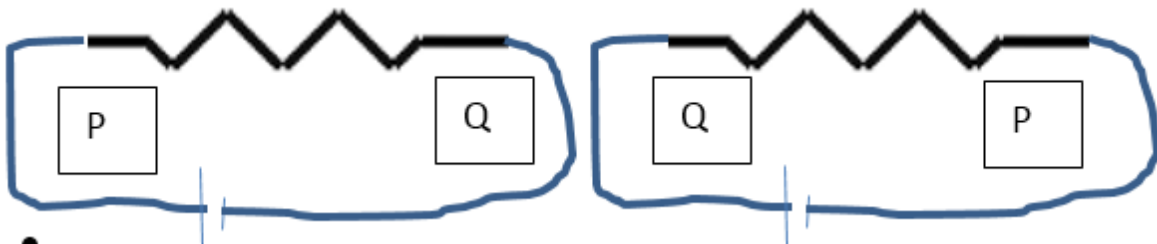
- **Wrapping a length of wire does not change its resistance.**
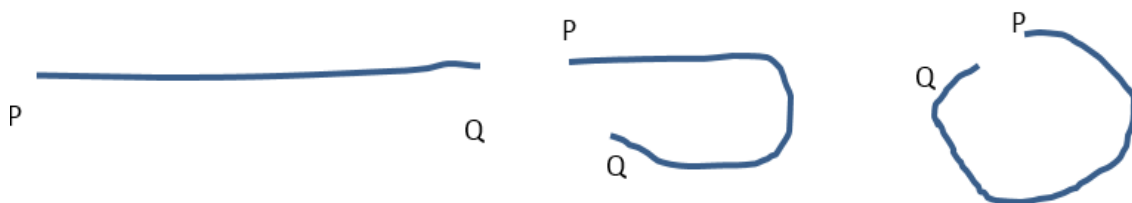
- The resistance changes if the two ends of a long wire are joined and placed in a circuit

It will be as though two wires of half of original length are connected in parallel

If we were to calculate its value, we would need rules of addition of resistances in parallel.

- Resistance of a wire of length (l) of homogeneous material (ρ) of uniform area (A) of cross section is given by

$$R = \rho\frac{l}{A}$$

- Different lengths of the same material and the same area of cross section wire will have different resistance.
- Wires of the same material have the same length but different areas of cross section have different resistance. specific wire gauge (s w g) is a specification for a wire based on its diameter
- Wires of the same length have the same area of cross section but different material will have different resistances.
- If the resistance wire of a certain length, is of uniform density but non uniform area of cross section
    - Its specific resistance remains the same
    - Its resistance remains the same
    - Its resistance per cm will change
- If a wire is stretched to increase its length
    - Its specific resistance remains the same as the material remains the same
    - Its resistance will change
    - Its volume will remain the same
    - Its diameter /area of cross section will decrease

Combinations Of Resistances

In many practical circuits in everyday life two or more resistances may be required to be combined. This is done to achieve a value of resistance which you may not have or they just have to be combined for any useful purpose.
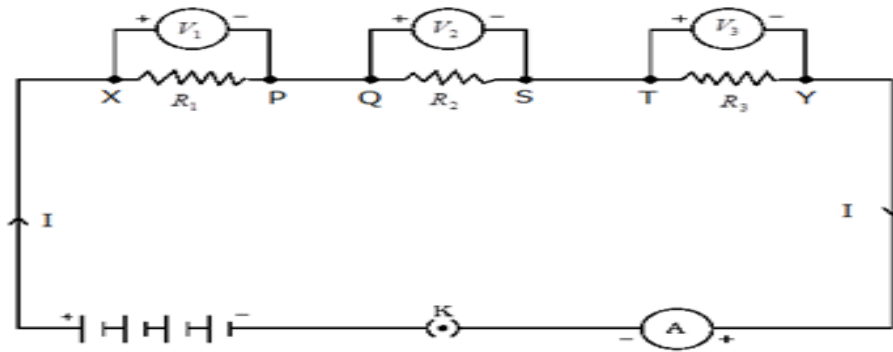
There are two ways to combine resistances

i) **In series**

ii) **In parallel**

let us understand what this means

Resistances in Series

In this combination the resistances are combined end to end

In the circuit diagram given, there are three resistances R_1 ,R_2 and R_3

Taking a closer look at the three resistances

A series combination of three resistors R1, R2, R3.

**Can we have two resistances in series?**

Can we have more than three resistances in series?

The characteristics of series combination are as follows

 i. **Same current flows through each resistor.**

 ii. **Potential drops, across different resistors, are different.**

   a) **Across $R_1 = I\, V_1$**

   b) **across $R_2 = I\, V_2$**

   c) **across $R_3 = I\, V_3$**

 iii. **The combined resistance is larger than the largest resistance in series**

Combined series resistance or equivalent resistance

The above circuit can be redrawn as
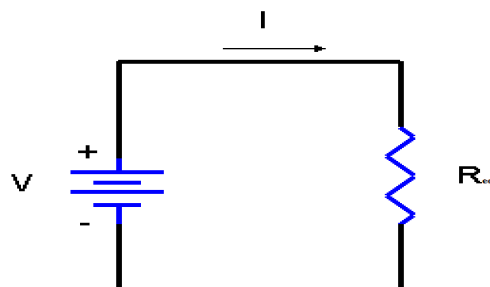
For the **equivalent** circuit we can write

$$V = IR_{eq}$$

The terminal, or end, through which current enters in a device, is treated as '+' end as it is at a higher end.
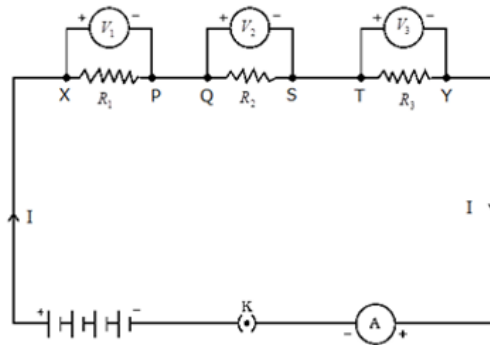
Let current I flow through each resistor in the circuit, and then net potential difference across the combination of the three resistors will be the sum of potential differences across each resistor.

Therefore
$$V = V_1 + V_2 + V_3$$
$$V = IR_1 + IR_2 + IR_3$$
$$V = I\left(R_1 + R_2 + R_3\right)$$
$$V / I = R_1 + R_2 + R_3$$

The expression for equivalent Resistance (R_{eq}) can be written as
$$\mathbf{R_{eq} = R_1 + R_2 + R_3}$$

For n resistances in series
$$R_{eq} = R_1 + R_2 + \ .... + R_n$$

The value of R_{eq} in a series combination is larger than the largest resistor of the group. Thus, we see that the equivalent resistance increases through a series combination of resistances.

Example

Suppose we have three resistances of 1Ω, 2Ω and 3Ω connected in series.What is the combined resistance?
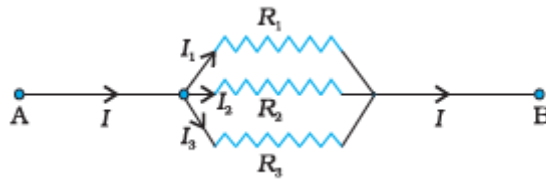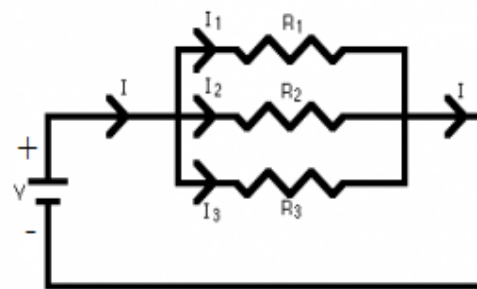
Solution

$\mathbf{R_{eq} = 1 + 2 + 3}$

$= 6\ \Omega$

Value of R_{eq} of 6 Ω will be greater than 3Ω.

Resistances in Parallel

If the three resistances R_1, R_2, R_3 were connected in parallel.
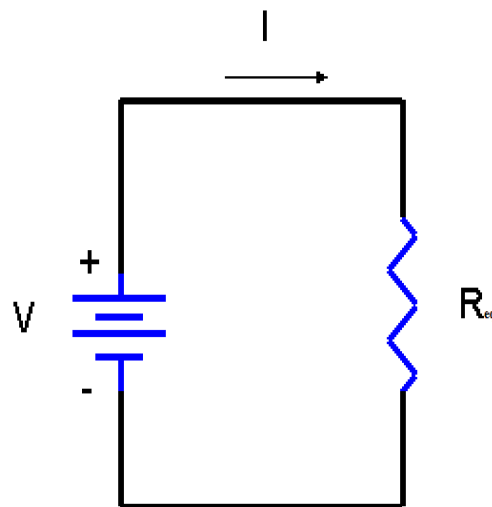

Potential difference across each branch will be the same .But the current in each will be

different and will depend upon the branch resistance.

The above circuit can be redrawn as

For the equivalent circuit we can write

$$V = I\ R_{eq}$$

The characteristics of parallel combination are as follows

i) **Same potential drop across each resistor.**

ii) **Different currents in different resistors of each branch**

Current in $R_1 = \dfrac{V}{R_1}$

Current in $R_2 = \dfrac{V}{R_2}$

Current in $R_3 = \dfrac{V}{R_3}$

iii) The combined resistance is lesser than the least branch resistance

Let potential difference V be applied across each resistor in the circuit with the help of cell. Then net current drawn from the cell will be the sum of currents through each resistor.

Therefore $\qquad\qquad\qquad\qquad I = I_1 + I_2 + I_3$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V\left\{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right\}$$

$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The expression for equivalent resistance (R_{eq}) can be written as

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- **The value of R_{eq} in parallel is less than the least resistance in the group.**

Example

3 resistances of 1Ω, 2Ω and 3Ω connected in parallel find the combined resistance.

Solution

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$$

$$\frac{1}{R_{eq}} = \frac{11}{6}$$

Req =6/11=0.54 Ω , which is less than the least resistance in any of the branches.

Significance of equivalent resistance

An equivalent resistance or resultant resistance is that single resistance, which draws the same current from the cell as is being drawn by the given combination of connected resistances, together from the same cell.

Need for Series and Parallel Resistance Combination

To make desired resistances we need to join available resistances in different types of arrangements. Also the complex arrangements of resistances can be simplified by identifying

resistances in series or parallel arrangements. Following points can help us to understand the basic need and difference between the series and parallel type of arrangements of resistances.

- In the series combination, the net or equivalent resistance increases; hence current drawn from the cell decreases. In parallel combination the net or equivalent resistance decreases; hence current drawn from the cell increases.

  Example: We have 3 resistances of 1Ω, 2Ω, 3Ω, and a battery of 12V. If maximum current is to be drawn from the cell, then resistances should be connected in parallel. To draw minimum current from the cell, resistances should be connected in series.

- In series combination, 'breakdown' of one device (resistance) stops the working of all the other devices (resistances).

  In a parallel combination, 'breakdown' of one device (resistance) does not affect the working of the other devices (resistances).

  Example: Whenever there are fluctuations in household voltage, the fuse may get burnt so that a heavy current is not passed to household devices. The 'breakdown' of the fuse is intended to save household devices. Hence, the fuse is put in series with the household circuit.

  In case of 'breakdown' one device should not affect the working of other devices. Hence different devices are connected in parallel in a household circuit.

Types of Resistors

There are mainly two types of resistors

- **Wire-wound resistors-** They are generally made from metal alloys; their thermal coefficient of resistance is very small and resistivity is very high. They are used in making 'precision resistances' used in laboratories. They are costly; hence they are not used for commercial purposes.

  Most commonly used material is 'Manganin', which is a**n alloy of 86% copper, 12% manganese, and 2% nickel.** It was first developed by Edward Weston in 1892, improving upon his earlier composition of Constantan.

  Constantan, also known as Eureka, is a copper–nickel alloy usually consisting of 55% copper and 45% nickel.

- **Carbon resistors**- These are made of carbon; they are low cost and are used for commercial purposes. Their range is quite high as compared to wire bound resistances.

  The value of the resistance of carbon resistors, is given by a color code.

Colour Coding of Carbon Resistance

There can be three or four colour bands.

a) **The first two colours give the first two digits in the resistance value.**

b) **The third colour represents the power of ten for the multiplier of the resistance value.**

c) **The last colour is the tolerance of the resistance value.**

Each resistor has three or four stripes or bands as shown in the figure given below.
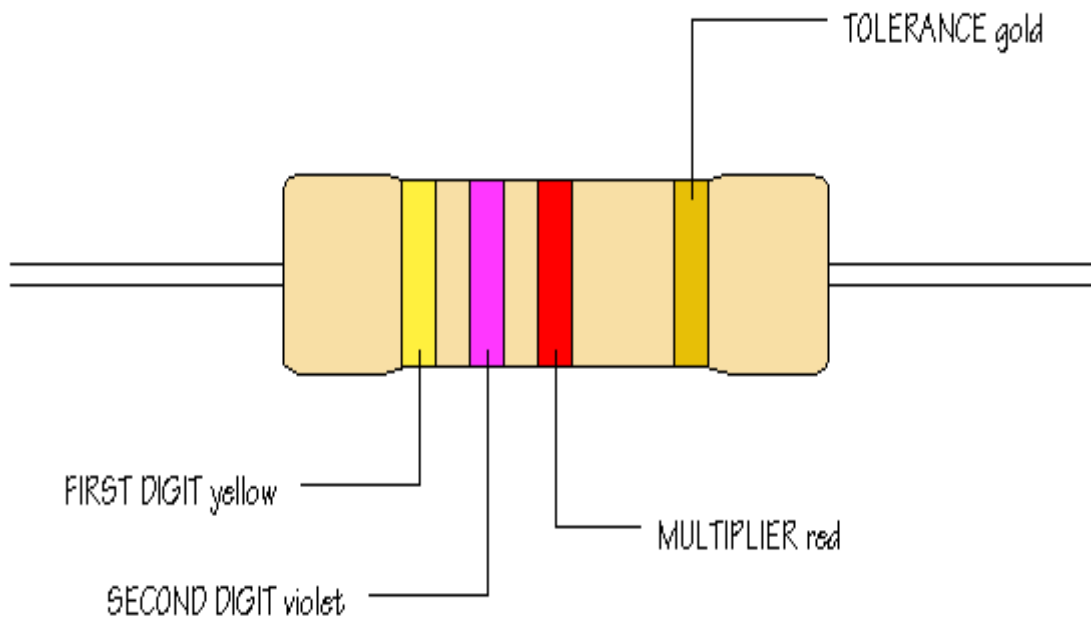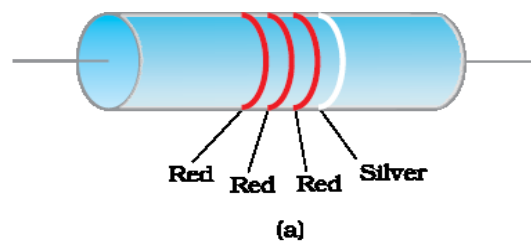

Tolerance indicates the order of likely variation in the value of resistance, value of resistance, as read from its 'colour code'. Following colour notations are used for depicting the tolerance.
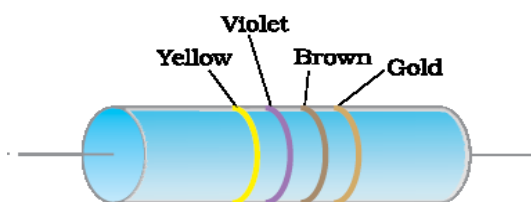
Gold = 5%

Silver = 10%

No color = 20%

Try These

The values and multiplier associated with each colour has been given in the resistor colour coding chart.

Colour	Number	Multiplier	Tolerance (
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

The colours and their values are given by following rhyme

blackB^0 lack brownB^1rown redR^2ed orangeO^3range yellowY^4ellow greenG^5ot blueB^6eautiful violetV^7 ery greyG^8ood whiteW^9ife

B B R O Y G B V G W

Example:
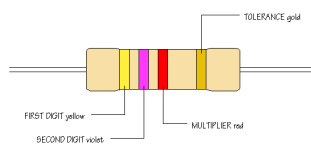
Solution

The value of colours mentioned in the above given figure is Yellow (4), Violet (7), Red (2), Gold (5%). Therefore the value of resistance is

$$(47 \times 10^2 \pm 5\%)\ \Omega \implies (4700 \pm 5\%)\ \Omega$$

$$\text{Or } (4700 \pm 235)\ \Omega$$

Therefore the value of resistance can be any anywhere from 4465Ω to 4935Ω.

Example

If the four colour bands are given as: Blue (6), Yellow (4), Green (10^5), Silver (10%) then its resistance is given as $64 \times 10^5 \pm 10\%$ Ω

Example

A resistance is marked as Green, Yellow, Gold. Find its value.

Solution

Green → 5

Yellow→4

Gold→10^{-1}

As only three bands are there therefore for tolerance no colour is used.

Tolerance (no colour) →20%

Therefore, the value of resistance is $(54 \times 10^{-1} \pm 20\%)$ Ω = $(54\pm20\%)$ Ω

Example

Give color code for a 6300Ω resistance.

Solution

6→Blue

3→orange

10^{-2}→red

Therefore, the color code is **Blue, Orange and red**

More Solved Examples

Example

How will you represent a resistance of 3700Ω $\pm$ 10% Ω by color code?

Solution

The value of carbon resistance = $3700 \pm 10\%$ Ω = $37\times10^2 \pm 10\%$ Ω

The colors assigned to numbers 3, 7 and 2 are **orange, violet and red** respectively.

For $\pm$10% accuracy, the color is silver. Thus, bands of colors on carbon resistance, in sequence, are orange, violet, red and silver.

Example

Four resistors of 12Ω, each are connected in parallel. Three such combinations are then connected in series. What is the total resistance? If a battery of 9V emf and negligible internal resistance is connected across the network of resistors, find the current flowing through each resistor.

Solution
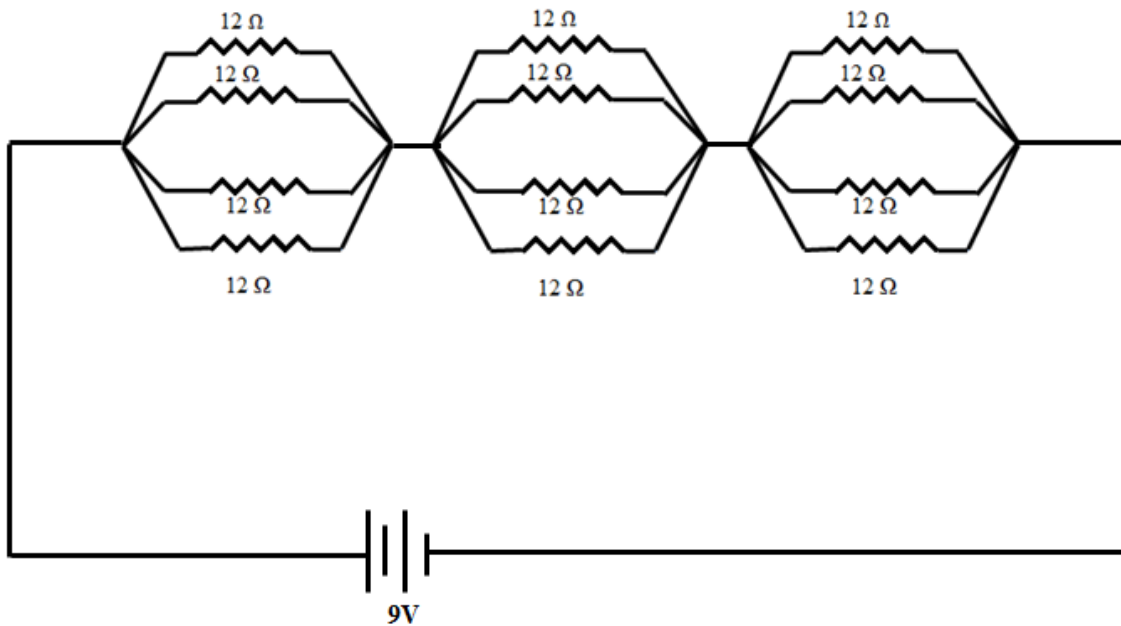
The network of resistors, connected to a battery of emf 9V is shown in figure above.

Let I be the total current in the circuit and R' be the effective resistance of four resistors of 12Ω each, in parallel,

Then, $1/R' = 1/12 + 1/12 + 1/12 + 1/12 = 4/12$

Therefore   **R' = 3Ω**

The total resistance of the network of resistors is then

$R = 3 + 3 + 3 = 9Ω$

Current in the circuit is $I = E/R = 9/9 = $ **1A**

Since all four resistors in parallel are of equal resistance.

So, the same current will flow through each resistor.

Therefore, current through each resistor

$i = I/4 = 1/4 = $ **0.25 A**

Example

What is the color of the third band of a coded resistor of resistance $2.3 \times 10^2 \, Ω$?

Solution

Resistance $= 2.3 \times 10^2$ Ω $= 23 \times 10^1$Ω. Therefore, the color of the third band of a coded resistance will be related to number 1. It is therefore brown.

Example

A resistor of 5 Ω is connected in series with a parallel combination of number of resistors each of 5Ω. If the total resistance of the combination is 6Ω, how many resistors are in parallel?

Solution

Let n resistors each of 5Ω be connected in parallel. Their effective resistance is

$1/R_p = 1/5 + 1/5 + 1/5 + \ldots$ n times $= n/5$ or $R_p = 5/n$

As this parallel combination of resistances is connected in series with 5Ω resistance, the total resistance of the combination is

$R = R_p + 5 = (5/n + 5)$ Ω $= 6$ Ω

$5/n = 6 - 5 = 1$Ω

n = 5

Questions For Practice

i) **Two wires X, Y have the same resistivity, but their cross-sectional areas are in the ratio 2:3 and lengths are in the ratio 1:2. They are first connected in series and then in parallel, to a d.c. source. Find out the ratio of drift speeds of the electron in the two wires for the two cases.**

ii) **Three resistors 1Ω, 2Ω and 3Ω are connected in series. What is the total resistance of the combination? (b) If the combination is connected to a battery of emf 12V and negligible internal resistance, obtain the potential drop across each resistor.**

iii) **Three resistors 2Ω, 4Ω and 5Ω are connected in parallel. What is the total resistance of the combination? (b) If the combination is connected to a battery of emf 20V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.**

iv) **Given n resistors each of resistance R. How will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?**

v) **Given the resistances of 1Ω, 2Ω, 3Ω, how will you combine them to get an equivalent resistance of**

(i) (1/3) Ω

(ii) (11/5) Ω

(iii) 6Ω

(iv) (6/11) Ω

Summary

- **Series Combination of Resistances**

  The expression for equivalent Resistance (R_{eq}) for resistances joined in series can be written as

  $$Req = R_1 + R_2 + R_3$$

  The value of R_{eq} in a series combination is larger than the largest resistor of the group.

- **Parallel Combination of Resistances**

  The expression for equivalent resistance (R_{eq}) for resistances joined in parallel can be written as

  $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

  The value of R_{eq} in parallel is less than the least resistance in the group.

- **An equivalent resistance or resultant resistance is that single resistance, which draws the same current from the cell as is being drawn by the given combination of connected resistances, together from the same cell.**